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Causality Testing using Higher Order Statistics

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Abstract : A new causality test based on Higher Order Statistics (HOS) is proposed in this paper. The test can be applied on non Gaussian time series. The methodological novelty is the usage of a two- step causality test based on digital whitening, which is performed by ARMA-HOS filters. To substantiate further the method an empirical analysis of the relationship between interest rate spread and real gross domestic product (GDP) growth is presented for the period 1981:q1 -2009:q1. Spread is measured as a difference between 10 years bond yields and three months Treasury bill rates in US. The fist step applies ARMA-HOS models to obtain white residuals from quarterly term structure (TS) and GDP growth. The second step tests dynamical correlation of TS and GDP growth residuals. Results show that the proposed test can capture the information about non Gaussian properties of the random variables being tested. The test is compared with Granger-Sims causality test. The paper questions reliability of the Granger test.

1. Introduction

The availability of large data sets of high frequency time series in finance and economics has led to the settlement of some old disputes regarding the nature of the data but it also generated a new challenges.

A set of properties common across many financial variables, instruments and markets, has been observed in independent studies and classified as “stylized facts”. One of the most important stylized properties of asset returns and financial variables in general , besides the absence of correlation, is heavy tails or existence of higher order moments and tail index which is finite and higher than two and less than six (Cont 2001).

The methodology widely used to test the occurrence of causality is known as Granger's methodology. Actually, Wiener was the first to state causality definition by suggesting that Xt is causal to YT if Xt reduces mean square prediction error of YT. Granger explored further Wiener's definition. Sims gave content to Granger's definition, by assuming that (Xt, YT) are jointly covariance stationary Gaussian processes and proving the causality theorem. The theorem states that for Xt and Yt having autoregressive representations, Yt can be expressed as distributed lag function of current and past Xt with a residuals that are not correlated with any values of Xt, past or future, if and only if Yt does not cause Xt in Granger's terms. Application of Granger-Sims methodology is usually used with two objectives: to test causality between different economic variables and simultaneously to define lags for which that causality exists. Therefore, while searching for lag identification, authors are forced to ignore the fact that residuals from the test models might be uncorrelated. Having realized shortcomings of the ad hock filter while applying Granger-Sims tests, Hough and Pierce (1977) introduced the causality test bases on the correlation between driving white noises for Xt and Yt, ut and vt respectfully. Although Box (1970) introduced the idea for the first time, this test has not brought a wide attention in econometrics.

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The empirical part of the paper test causality between GDP growth and Term Spread.
In fact, over the last decade empirical research has demonstrated positive relationship between the slope of the yield curve and real economic growth. The predictive power of the term spread has been recognized beyond academic research arena. The conference board uses the yield spread in constructing its Index of Leading Indicators. The fact that yield curve slope changes across the business circle is the cause of research that investigate recession and power of the term spread to predicate it. The slope of the term structure has been often represented in the economic literature as the spread between long term bonds and short term treasury bills.
The first papers, dealing with US data, found significant relationship between the term structure spread and real activity with lead times between 1 to 8 quarters (Chen (1991), Estrella (1991), Harvey (1995), Dotsey (1998), Bonser (1977), Ang (2003) .Guided from the intuition that during recessions, upward sloping yield curves indicate bed times today, but also better times tomorrow, researchers predicted GDP growth using LS regression. Bonser-Neal further established investigated at what horizons does the yield spread best aid in predicating real growth.
On the other side a cause of the possible relationship between term structure and GDP growth according to Taylor (1993) is monetary policy reaction function. His model contains Philips curve, dynamic IS curve, Fisher equation the expectations hypothesis and monetary policy rule. Estrella explored the model and found positive relationship between the spread and GDP growth.
Although the results obtained for different periods show strong relationship between Term spread and GDP growth they also demonstrate that relationship might not be stable over time.

The aim of this paper is to propose and to apply HOS based causality test to investigate dynamical relationship between term spread and real GDP growth. The novelty of the paper is the two step HOS based test bases on the assumption that possible cause of the instability of the relationship are non Gaussian properties of the variables that can be captured by higher order cumulants. In the first step two time series are whitened using time series models (ARIMA models based on higher order cumulants) in order to obtain prediction errors known as innovations. In the second step causality between white innovations is performed using Pierce & Hough test. This test appeared to be useful in eliminating potential influence of a third, unknown variable and appreciating the fact that Xt might not be the only variable that explains Yt.
To sustain theoretical analysis, the first part of the empirical analysis is done with US Term Spread (TS) data and real GDP quarterly data, for the period 1988: q1 to 2009:q1.

The paper is organized as follows: The second section provides causality tests used in literature so far. The third section describes the HOS based test. The fourth section contains statistical data description and empirical results obtained using HOS test. The last section contains conclusion.

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2. Problem formulation and methodology

2.1 Granger-Sims Causality Test

The most popular method for testing statistical causality between stock prices and the economy is “Granger-causality" test proposed by C.J.Granger (1969). According to Granger, X causes Y if the past values of X can be used to predict Y more accurately than simply using the past values of Y. In other words, if past values of X statistically improve the prediction of Y, then we can conclude that "Granger-causes" Y. If the sum of the squared residuals that remain after getting econometric model between Yt and is Xt denoted by SSR the test gets the form:

SSR0 $(Y_t /(Y_{t-1} + X_{t-1}) < SSR1\ (Y_t / Y_{t-1})$, if X_t Granger causes Y_t
To compare two variances F test is to be used .It should be pointed out that given the controversy surrounding the Granger causality method, the empirical results and conclusions drawn from them should be considered as suggestive rather than absolute. This is especially important in light of the "false signals" that the test has generated in the past.

2.2 Box -Hough test

As it was theoretically proven in the literature, alternative causality test is based on whitening filtration of X_t and Y_t, or by testing " white " residuals of the both variables X_t and Y_t. This test is supposed to eliminate a possibility of having relationship between two variables when both are driven, or influenced by some third variable. Recent economic research presented in the literature, unfortunately is not based on this method perhaps since it appears to be rather complicated for traditional researchers.
It was proven by Hough (1977) that if there is dynamical correlation between Y_t prediction errors and past X_t prediction errors we can say that X_t drives or causes Y_t. Vice-versa, if there is dynamical correlation between Yt prediction errors and past X_t prediction errors we can say that X_t drives Y_t.If prediction errors of X_t drive Y_t and prediction errors of Y_t drives X_t, there is feedback between two variables.

Formally Pierce and Haugh have defined causality restrictions regarding correlation coefficient ρ_{uv} between driving white noises for Y_t and X_t, $\mathbf{u_t}$ and $\mathbf{v_t}$:

$\rho_{uv)}$**(k)>< 0**	For every **k>0**	X_t **causes Y**
ρ_{uv} **(k)> < 0**	For every **k<0**	Y_t **causes** X_t
ρ_{uv} **(0)> < 0**		**Instantaneous Causality**

Later on, Box and Haugh (1977) proved that ρ_{uv} has asymptotically normal distribution with variance 1/(n-k), where n is the number of observations and thus enabled causality testing and k being the lag size.

The rational behind this test might be explained by two facts: Dynamical cross correlation between two stationary variables gives false signals about relationship if transfer functions of the ARMA models used to describe X_t and Y_t are linked; White residuals, from ARMA models, have one more meaning: one step ahead prediction errors for X_t and Y_t, or innovation. Therefore one can say that X_t causes Y_t if X_t innovations cause Y_t innovation.

3. HOS based test

Let X_t and Y_t be jointly stationary non Gausian processes with finite first and second , third and forth moments that can be treated as outputs from the linear ARIMA filters, whose inputs are white noise signals: $\mathbf{u_t}$ and $\mathbf{v_t}$ respectively:

$$\mathbf{A1(Z)^*\ DX_t= B1(Z)^*\ u_t} \qquad (1)$$

$$\mathbf{A2(Z)^*\ DY_t= B2(Z)^*\ v_t} \qquad (2)$$

Where **Z** is a backward shift operator : $\mathbf{Y_{t-1}=ZY_t}$ **,** $\mathbf{Y_{t-k}=Z^kY_t}$, $\mathbf{A(Z) = 1\text{-}\alpha_1Z\text{-}\alpha_2Z^2 - \ldots\alpha_pZ^p}$ **and** $\mathbf{B(Z) = 1\text{-}\beta_1Z\text{-}\beta_2Z^2 - \ldots\beta_qZ^q}$ are AR and MA filters of orders p and q respectively , **D** is the first difference filter, $\mathbf{DY_t = Y_t - Y_{t-1}}$, $\mathbf{D^k}Y_t\mathbf{=Y_t - Y_{t-k}}$
It is worth stressing that the main premises in this methodology is that each stationary time series is treated as output from AR(p), MA(q) or ARIMA(p,q) filter , which has as input uncorrelated and non Gaussian shocks known as "non Gaussian white noise".

Given time series X_t and Y_t observed at regular sampling interval it is necessary to define the relationship between them: as X_t causes Y_t , Y_t causes X_t , feedback or independence. The empirical research problem in this paper is to identify relationship between TS and percent GDP growth .

It was proven by Haugh (1977) that if there is dynamical correlation between Yt prediction errors and past X_t prediction errors we could say that X_t drives or causes Y_t. Vice-versa, if there is dynamical correlation between Yt prediction errors and past X_t prediction errors we can say that X_t drives Y_t.If prediction errors of X_t drive Y_t and prediction errors of Y_t drives X_t, there is feedback between two variables.

In this article time series modeling is done using HOS-ARIMA (p,q) models based on higher order cumulants .The later type of the model is used since it was found that ignoring non Gaussian nature of both time series significantly reduce the power of the causality test.

3.1. ARMA parameter estimation using cumulants

A new method of parameter estimation for non Gaussian processes ,using cumulants follows Yule-Walker system where autocorrelations are replaced by third or fourth order cumulants, Giannakis (1987) , was the first to show that AR parameters of non-Gaussian ARMA digital signals can be calculated using the third- and fourth-order cumulants of the output time series given by:

$C^3_x(\tau 1,\tau 2)= (\sum(x(t)x(t+\tau_1)x(t+\tau_2))/n,$ (3)

$C^4(\tau 1,\tau 2,\tau 3,)= (\sum(x(t)x(t+\tau_1)x(t+\tau_2)\, x(t+\tau_3))/n -$

$-C^2_x(\tau 1)\, C_x(\tau 2-\tau 3) - C^2_x(\tau 2)\, C_x(\tau 3-\tau 1)-C^2_x(\tau 3)\, C_x(\tau 1-\tau 2),$ (4)

where n is a number of observations and where the second-order cumulant $C^2_x(\tau)$ is just the autocorrelation function of the time series x_t.
The zero lag cumulant of the order3 $C^3_x(0,0)$ normalized by σ_x^3 is skewness γ^3_x; $C^4_x(0,0,0)$ normalized by σ_x^4 is known as kurtosis γ^4_x.
A new method of the AR parameter estimation for non-Gaussian ARMA (p,q) processes is based on the modified Yule-Walker system where autocorrelations are replaced by third or fourth order cumulants (Gianninakis -1990):

$$\sum_{1=1}^{p} \alpha i\, C^3(k-i,k-l) = - C^3(k,k-l) \qquad k\geq l\geq q+1 \qquad (5)$$

$$\sum_{1=1}^{p} \alpha i\, C^4(k-i,k-l, k-m) = - C^4(k,k-l, k-m) \qquad k\geq l\geq m\geq q+1 \qquad (6)$$

The efficient MA parameter estimation can be performed by applying one of the proposed algorithms, for instance, q-slice algorithm (Swami 1989). Q –slice algorithm uses autoregressive residuals calculated after estimating the AR parameters or ARMA (7). Following up, the impulse response parameters ψ_I of the pure MA model are then estimated using cumulants (8):

$$x_t=\sum_{0}^{\infty}\psi_j a_{t-j} \qquad i=1.2\ldots\infty \qquad (7)$$

$$\psi j = \frac{\sum^{p} \alpha i\, C^3(q-i,j)}{\sum^{p}\alpha i\, C^3(q-i,0)} \qquad j=1,2\ldots q \qquad (8)$$

Or by using :

$$\psi j = \frac{\sum^{p} \alpha i\, C^4(q-i,j,0)}{\sum^{p}\alpha i\, C^4(q-i,0,0)} \qquad j=1,2\ldots q \qquad (9)$$

The MA parameters of the ARMA model are obtained by means of the well known relationship

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$$\beta_j = \sum_{i=1}^{p} \alpha_i \, \psi_{(j-i)} \quad j=1,2\ldots q \qquad (10)$$

The cumulants based ARMA estimates are shown to be asymptotically optimal by Friendler B. and Porat B. (1989). It is important to underline that the method explained above has been used so far only in the area of digital signal processing and has not been used in finance and economics.

Empirical Results

Granger test results

Real GDP data are taken from Bloomberg, 10 years treasury bonds and three months treasury bills rates are taken quarterly from the web page economagic.com for the period 1981:q1 :2009 :q1. Figure 1 shows how all variables change .

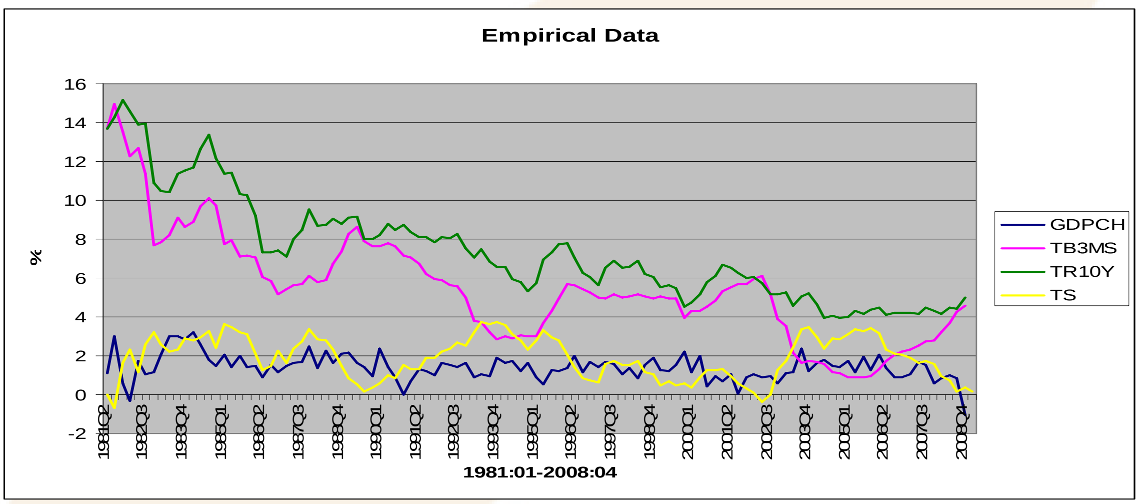

Figure 1 : GDP and interest rate yields

Statistical data description is obtained using E-Views program and it is presented in the table 1.

Table 1

	TSPREAD	GDPCH
Mean	1.910	0.014
Median	1.910	0.014
Maximum	3.730	0.032
Minimum	-0.670	-0.010
Std. Dev.	1.086	0.007
Skewness	-0.181	-0.021
Kurtosis	2.029	5.790
Jarque-Bera	4.970	14.824
Probability	0.083	0.001
Observations	111	111

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The skewness and kurthosis factors, which are given in the table 1, show that both variables are non-Gaussian (according to the skewness, kurtosis and the Jarque-Bera test for normality).

The results of the Granger causality test between the GDP percent change data and Term spread (TS) for the lags 1,2…8 are presented in Table 2. The test shows a feedback relationship between Term Spread and GDP change for the quarters 1 and 2.It also shows that term spread does Granger cause GDP change across three quarters ,while GDP change Granger causes term spread over next two quarters.

Table 2: Granger Causality Test results

	Sample: 1981Q1 2009Q1			
Lags	Null Hypothesis:	Obs	F-Statistic	Probability
1	TSPREAD does not Granger Cause GDPCH	110	**7.29281**	0.00805
	GDPCH does not Granger Cause TSPREAD		**4.60339**	0.03417
2	TSPREAD does not Granger Cause GDPCH	109	**6.24964**	0.00273
	GDPCH does not Granger Cause TSPREAD		**4.35234**	0.0153
3	TSPREAD does not Granger Cause GDPCH	108	**5.23773**	0.00212
	GDPCH does not Granger Cause TSPREAD		1.66559	0.17919
4	TSPREAD does not Granger Cause GDPCH	107	3.05684	0.02025
	GDPCH does not Granger Cause TSPREAD		1.22396	0.30564
5	TSPREAD does not Granger Cause GDPCH	106	1.73825	0.13331
	GDPCH does not Granger Cause TSPREAD		0.68063	0.63919
6	TSPREAD does not Granger Cause GDPCH	105	1.75143	0.11794
	GDPCH does not Granger Cause TSPREAD		0.20243	0.97524
7	TSPREAD does not Granger Cause GDPCH	104	2.22299	0.03955
	GDPCH does not Granger Cause TSPREAD		0.68615	0.68342
8	TSPREAD does not Granger Cause GDPCH	103	1.58693	0.14056
	GDPCH does not Granger Cause TSPREAD		1.30233	0.25324

HOS based causality test results

The HOS based test ,proposed in this article, is based on digital whitening . Residuals From GDP change and Term Structure data are obtained by using higher order moments as explained above. The best ARMA model for GDP change is found to be ARMA(4,4) . The model parameters (table 3) are estimated using fourth order cumulants .and MATLAB toolbox HOSA , developed by Swami (1992) . Likewise, the best model for Term spread appeared to be AR(1,4) model which is presented in the table 4.

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Table 3: GDP changes
ARMA-HOS model

Variable	Coefficient	Std. Error	t-Statistic
C	1.38106	0.146811	9.407083
AR(1)	0.128982	0.068527	1.88222
AR(2)	0.143497	0.080827	1.775364
AR(3)	0.171894	0.055231	3.11225
AR(4)	-0.046561	0.010904	-4.27013
MA(1)	0.274197	0.046255	5.927944
MA(2)	0.242425	0.121984	1.98735
MA(3)	-0.081094	0.018375	-4.41332
MA(4)	0.268694	0.048755	5.51115

Table 4: Term Spread changes
ARMA-HOS model

Variable	Coefficient	Std. Error	t-Statistic
C	1.82815	0.282767	6.46522
AR(1)	0.991982	0.051261	19.35151
AR(4)	-0.151676	0.046951	-3.23052

GDP change and TS cumulants are calculated using equations (3) and (4) and are presented on the figures 2 and figure 4, while cumulants related to obtained residuals are presented on the figures 3 and 5.
The test states: If there is statistically significant dynamical relationship between current GDP residuals and past TS residuals TS causes GDP; If there is statistically significant dynamical relationship between current TS residuals and past GDP residuals GDP causes TS . If both hypotheses cannot be rejected, than there is a feedback relationship between TS and GDP.

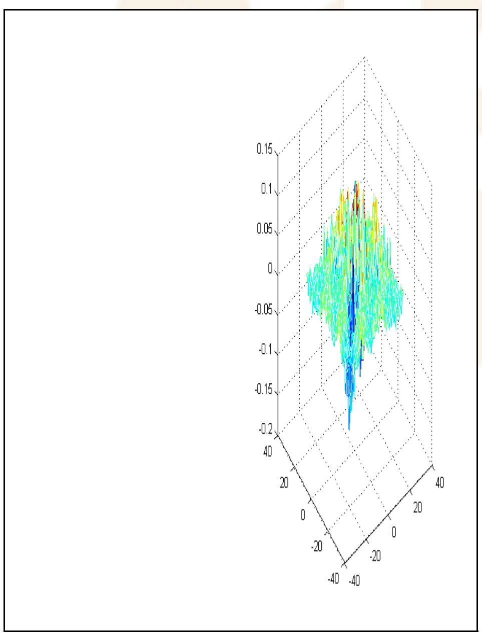

Fig.2 : GDP change- cumulants

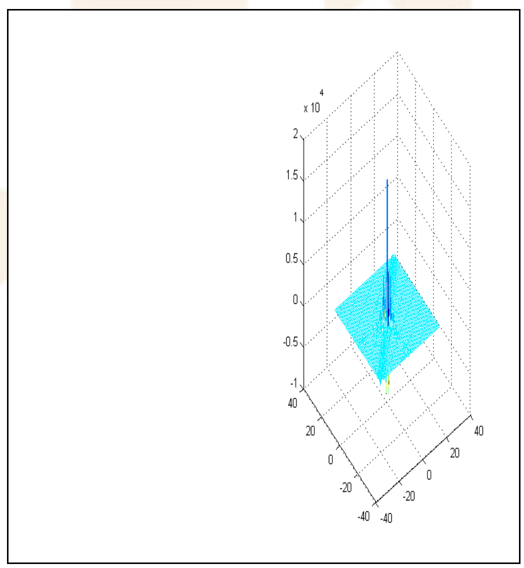

Fig.3 : GDP change-residual cumulants

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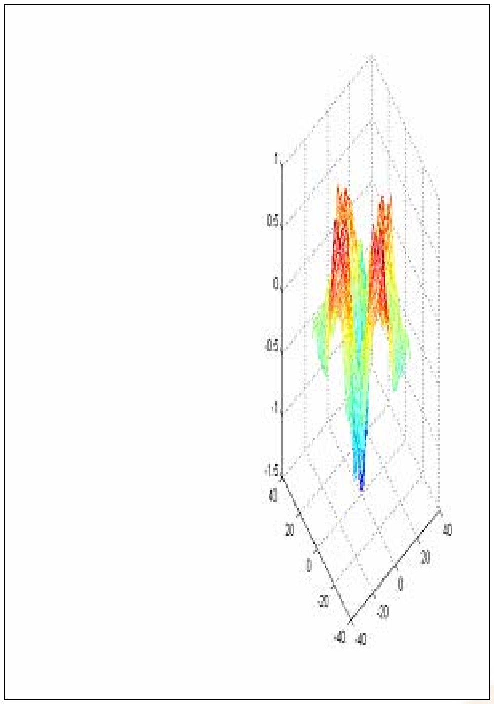

Fig.4 : Term Spread change- cumulants

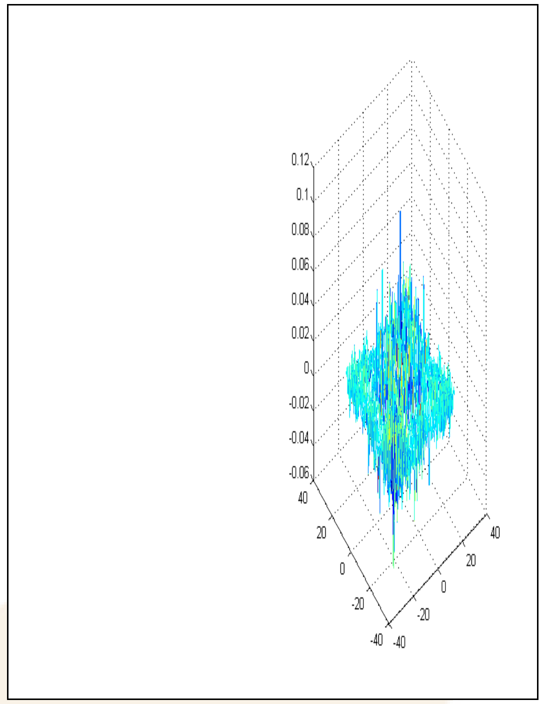

Fig.5 : Term Spread residual cumulants

Table 3 : HOS test results

Dependent Variable: RESGDP				
Method:HOS				
Included observations: 108 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	F
RESTS	-0.061	0.118	-0.517	0.267
RESTS(-1)	0.021	0.113	0.182	0.033
RESTS(-2)	0.298	0.108	2.745	7.533
RESTS(-3)	0.056	0.113	0.499	0.249
RESTS(-4)	0.012	0.004	2.812	4.012
RESTS(-5)	-0.092	0.113	-0.814	0.663
RESTS(-6)	0.224	0.108	2.078	4.319
RESTS(-7)	-0.069	0.109	-0.638	0.913
RESTS(-8)	-0.097	0.108	-0.899	0.663

Above results strongly prove that innovations or prediction errors of Term Spread cause innovations of percent changes of the real US GDP for the lags 2 ,4 and 6. For all the other lags F test shows a non significant causality

Conclusion

A new causality test based on HOS (Higher Order Statistics) is presented in the paper.
The paper further provides two theoretical contributions. Firstly, the proposed test solves the problem of "spurious causality "as a result of the wrong model order selection based on second order moments ,which than necessary leads to colored residuals and the wrong causality lag . The second theoretical contribution is achieved by using higher order cumulants to estimate model parameters and capture non Gaussian properties of the original time series.

To substantiate the analysis HOS base test was applied to test causality between Term Spread and real GDP dat in US for the period 1981:q1 -2009:q1. Obtained results clearly show that interest rate spread significantly influences GDP growth in the second, fourth and sixth quarters.
However, percentage of the explanation of GDP growth variability achieved by using term structure as explanatory variable in the last two decades is much lower that it was shown in the literature for the period 1970-1990.
There are two possible reasons for this finding: Granger causality test overestimate coefficient of determination due to wrong model order or most probably the same test doesn't capture higher order moments of the variables that are statistically related. As demonstrated in this paper, non Gaussian properties of the related variables are captured by the proposed ARMA –HOS test.

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